Use of space–time coding in coherent polarization-multiplexed systems suffering from polarization-dependent loss

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We evaluate the advantage of using space–time coding in order to increase the tolerance of fiber-optic communications systems to polarization-dependent loss (PDL). Focusing on three particular codes, the Golden Code, the Silver Code (SC), and the Alamouti Code (AC), we calculate the amount of average PDL that can be tolerated for a given signal-to-noise ratio margin that is designed into the system. The SC is shown to be optimal in the case of low to moderate PDL, whereas, in the case of extreme PDL, the AC shows the best performance.

OCIS codes: 060.2330, 060.1660.

One of the most attractive features of coherent optical communications is that manipulation of the entire optical signal (phase and amplitude) can be performed in the electronic domain. As a result, impairments that are caused by unitary processes, such as chromatic dispersion or polarization mode dispersion, can be eliminated by the use of electronic signal processing [1]. Polarization-dependent loss (PDL), on the other hand, is a nonunitary phenomenon and, therefore, it cannot be compensated at the receiver, even in principle. The PDL of a system results from the accumulated contributions of a large number of inline elements that are present along the optical link. Since the polarization states and relative orientations of these elements are random and time dependent, the overall instantaneous PDL changes randomly, both in frequency and in time [2]. The main consequence of this situation is a corresponding randomization of the received signal-to-noise ratio (SNR) [3,4]. To deal with PDL, designers allocate an SNR margin. In other words, the system is designed to operate with a higher SNR than what is needed in the absence of PDL so that the probability that a system outage occurs is maintained below the commonly quoted value of 4×10^{-5} (equivalent to 20 min per year). An SNR margin of 1 dB that is designed into a coherent polarization-multiplexed system is known to buy immunity to as little as 1 dB of average PDL when optimized detection is used. In many situations, this reality puts significant stress on component makers and system designers. The problem of increasing the tolerance of fiber-optic systems to PDL is therefore a subject of utmost importance [5].

In this work we examine the advantage that can be gained from the use of space–time codes in terms of the tolerance of systems to PDL. We focus in particular on two types of codes; the Golden Code (GC) [6] and the Silver Code (SC) [7], which have been recently attracting significant attention in wireless applications [8]. We expand on the results of [9,10], with the main difference being that, unlike in [9,10], where PDL was modeled as a constant parameter, we incorporate the codes into a full model of PDL in which the actual statistics of this phenomenon is explicitly taken into account. By doing so, we are able to quantify the advantage in the use of such codes in terms of the outage parameters and, in particular, in terms of the amount of average PDL that can be tolerated by the system for a given SNR margin that is allocated for this phenomenon. We find that both the GC and the SC are very well suited to deal with the most relevant range of PDL values (between 0 and 6 dB of average PDL), in which they yield similar results. In the range of low to moderate PDL values (0 to 5 dB of average PDL), these codes closely follow the curve that describes the PDL dependence of the information capacity, with the SC having a small advantage. For comparison, we also present the results of the 2×2 version of the Alamouti Code (AC), whose main advantage is in its simplicity. As expected, the AC is found to be irrelevant in the range of low PDL values, since it incurs an inherent 3 dB loss in transmission rate. Nonetheless, we find it interesting that, in the case of very large PDL, the AC performs notably better than the other two codes.

The general idea of space–time coding [11] is that, instead of transmitting two mutually independent streams of data in the two polarization channels, controlled dependencies between the symbols are introduced. These dependencies reduce the sensitivity of the system to nonunitary distortions, such as those generated by the effect of PDL. Both the GC and the SC can be categorized as 2×2 space–time block codes. The transformations that are used to construct the SC and GC are described in [6,7]. They both map a, b, c, and d, four data symbols from a QAM constellation, into four alternative complex numbers, p_1, q_1, p_2, and q_2.

The symbols p_1 and q_1 are transmitted in two consecutive time slots over one polarization channel, whereas p_2 and q_2 are transmitted over the other polarization channel. Notice that, since four complex data values are simply replaced by four different complex values (of the same square average), the GC and SC encoding introduces no reduction in transmission rate.

The AC scheme is the simplest of the three and is given by the relations $p_1 = a$, $p_2 = b$, $q_1 = -a^*$, and $q_2 = b^*$. It maps two complex input values into four symbols, implying an inherent loss in the transmission rate. To compensate for this, the Alamouti-based scheme must transmit a

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more refined constellation consisting of M^2 constellation points, as opposed to M in the prior two cases. As we show in what follows, the penalty that is caused by the rate loss, which in our setting is approximately 3 dB, is more than compensated for when the average PDL is large. As always, a major issue when considering coded communications is the decoding complexity, and that is because all four symbols need to be decoded simultaneously for optimal detection. This reality is, however, not flagrantly inconsistent with the complexity of recently demonstrated optical receivers. Owing to their efficiency in battling the effects of fading, the GC and the SC are considered to be leading candidates in the most recent wireless communications standards.

The results presented throughout this Letter were obtained using the numerical procedure whose details are presented in [12], with the optical link represented as consisting of ten statistically independent PDL sections and with the local PDL vector of each section being Gaussian distributed. This has been known to be the most natural assumption regarding the PDL distribution [2,13], in the absence of more specific knowledge about the structure of the optical link [14]. A total of 10^6 link realizations were performed for every combination of link parameters.

The codes are implemented into the system according to the schematic illustrated in Fig. 1. The two separate streams of data are each encoded independently using a standard forward error correcting (FEC) encoder. We refer to this stage as outer encoding. Then, the MIMO code is applied in blocks of four symbols, two from each information channel, generating the encoded symbols that are transmitted over the two polarizations in the fiber. The receiver consists of the complementary decoding elements. Namely, the signals first go through the MIMO decoder and, after that, the outer FEC decoding is applied. In order for the scheme to operate properly, the bit-error rate (BER) that follows the MIMO decoder must be lower than the BER threshold of the outer FEC. Typical BER threshold values of outer FEC codes that are currently deployed in fiber-optic systems are between 10^{-3} and 10^{-4}. A convenient way of quantifying the effect of PDL on system performance is to define an equivalent SNR penalty parameter [12,15] η that is equal to the factor by which the SNR needs to be increased in order to compensate for the penalty induced by a given realization of the PDL in the fiber link. Since the PDL of the link changes randomly, so does the η parameter, and therefore it must be represented in statistical terms. Figure 2 shows the complementary cumulated distribution of η, i.e., the probability that η exceeds the value indicated by the horizontal axis in the figure. Of particular interest is the outage value of the SNR penalty η_{out}, which is the value of η that is exceeded with probability equal to, or smaller than, the allowed outage probability of the system $P_{\mathrm{out}} = 4 \times 10^{-5}$. The outage SNR penalty η_{out} is equal by definition to the system SNR margin that needs to be allocated in order to avoid outages with the probability of $1 - P_{\mathrm{out}}$. The distributions in Fig. 2, as well as in the rest of this Letter, were calculated for the quadrature phase-shift keying (QPSK) constellation in each polarization channel, and the baseline SNR was selected such that the raw BER (before FEC decoding) was 10^{-3}, consistent with the typical range of available FEC thresholds. The solid and the dashed curves correspond to the GC and to the SC, respectively. The blue dashed–dotted curves in Figs. 2(b)–2(d) represents the AC. Evidently, the SC has a small advantage at small average PDL values, but, at larger PDL, it is the GC that assumes a small advantage. The red solid curve corresponding to the result without space–time coding differs very significantly from the other two. Naturally, the AC is inferior to the two other codes at small PDL values, where the rate loss that is inherent to its operation is significant. Yet, in the limit of large PDL, it matches the performance of the other two codes and eventually exceeds it. Figure 3 summarizes a large number of simulations of the type described in the context of Fig. 2. The horizontal axis in Fig. 3 is the average PDL of the link, and the vertical axis shows the SNR that is required in order for the raw BER of the system to remain lower than 10^{-3} (with probability of $1 - P_{\mathrm{out}}$, where $P_{\mathrm{out}} = 4 \times 10^{-5}$). Once again, the solid and the dashed curves represent the result of the GC and the SC, respectively, whereas the dashed–dotted curve

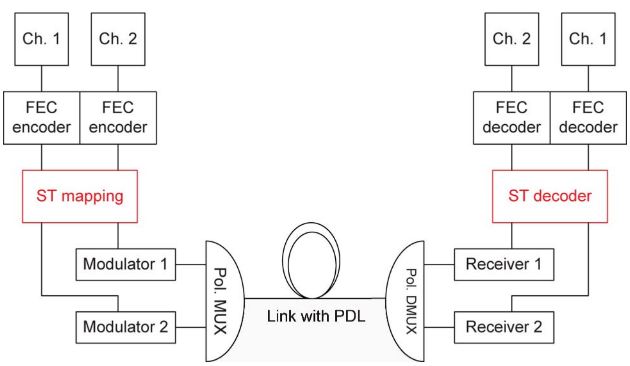

Fig. 1. (Color online) Schematic of coding implementation. ST stands for space–time.

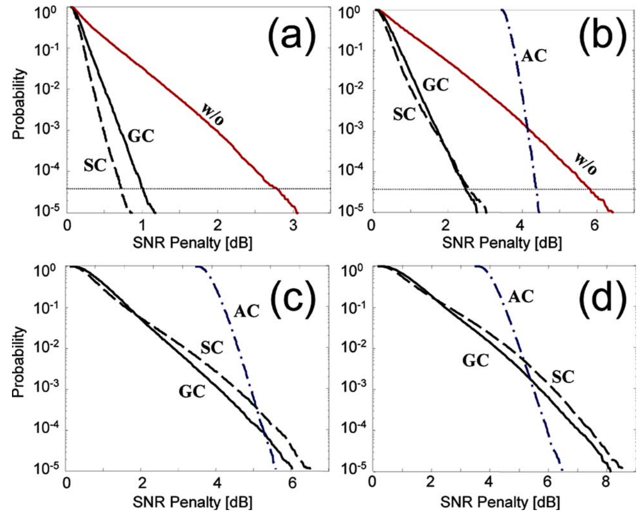

Fig. 2. (Color online) Complementary cumulated distribution of the PDL-induced SNR penalty for average PDL of (a) 3, (b) 5, (c) 8, and (d) 10 dB; w/o refers to the calculation without space–time coding. Dotted horizontal line indicates the outage probability of 4×10^{-5}. The curve corresponding to the AC is not shown in the 3 dB case, as it is almost outside the range of plotted values. Similarly, the curve without ST coding is outside the range of displayed values in parts (c) and (d). All curves were obtained for QPSK constellations.

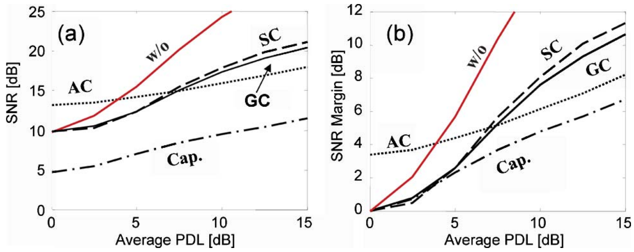

Fig. 3. (Color online) (a) SNR that is required to achieve a raw BER of 10^{-3} with probability $1 - P_{\text{outage}}$. The curve labeled Cap. shows the SNR that is needed in order to guarantee that the channel capacity exceeds 2 bits per polarization with probability $1 - P_{\text{outage}}$. (b) The SNR margin that needs to be allocated for the various codes.

shows the result of the AC. The result without space–time coding is marked by w/o. Also shown in the figure is the SNR that is required for the Shannon capacity of the PDL impaired channel to exceed 2 bits per symbol in each polarization channel (4 bits altogether), similar to the original QPSK constellation. The outage capacity was defined and discussed in the context of PDL in [16] and is equal to the capacity (mutual information between the input and output of the channel, assuming an isotropic Gaussian input) value that is exceeded with the probability of $1 - p_{\text{out}}$ [17]. The gap between the capacity curve and the actual performance curves with and without the GC is known as the gap to capacity and, as can be seen (from the case of 0 PDL), it is caused, among other reasons, by the imperfection of the outer FEC code. The system SNR margin that needs to be allocated for PDL is obtained by subtracting from the curves in Fig. 3(a) the baseline SNR value (namely, the SNR value that corresponds to PDL $= 0$). As is evident in the figure, at low PDL values, the slope of the curves representing the the GC and SC is very similar to that of the capacity curve, providing an idea on the optimality of those codes. For average PDL values in the range between 1 and 4.5 dB, the margin curve in the SC case is even slightly lower than that of the capacity-based curve, demonstrating a slight reduction in the gap to capacity. On the other hand, consistent with Fig. 2, the AC, which is strongly inferior to the other codes at low to average PDL, becomes the best code at high PDL values. Intuitively, this can be understood owing to the inherent ability of the AC to deal with the complete fading of one of the two channels, which is analogous to its ability to decode with one receive antenna.

We have considered the benefits that can be obtained from the use of space–time coding for increasing the PDL tolerance of fiber-optic systems. By accurately modeling the statistics of PDL, we quantified the benefit of coding in terms of the tolerable PDL for a given SNR margin. For example, with an SNR margin of 1 dB, the PDL tolerance can be increased from 1 to approximately 3.5 dB by the use of the considered codes, while bigger gains can be realized when a larger margin is allocated. The best performing code in the low average PDL regime was found to be the SC, with the GC performance lagging slightly. The AC was found to perform best in the very large PDL limit.

References and Notes

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